Symmetrical N-Port Waveguide Junction Loaded with Dielectric Sleeve and Metallic Post

S. P. Yeo, *Member, IEEE,* L. Qiao, and M. Cheng

***Abstract*—The present paper describes the development of a computer model that is able to predict the characteristics of the symmetrical N-port waveguide junction, which has a dielectric sleeve and metallic post inserted concentrically into its central cavity. Computational and experimental test results demonstrate that the resultant software package (which is compact enough to be run on an IBM 486 PC) can yield accuracies of ±0.5% for the scattering parameters of the junction.**

I. Introduction

Waveguide junctions are used extensively in microwave engineering. Although the more commonly available ones are of two-, three- or four-port construction, other novel N-port waveguide structures with $N \geq 5$ have also been proposed in the literature (e.g., those recently reported in [1]–[8]) for various special applications.

Of interest in the present paper is the symmetrical N-port waveguide junction depicted in Fig. 1, which has been used as a power divider/combiner [3] or as building blocks for a diversity of six-port reflectometer configurations [4]–[7]. Such junctions are not easy to analyze—especially when N is large—and some researchers [3]–[5] have resorted to empirical cut-and-try design procedures, which unfortunately are labor-intensive as well as time-consuming. With the ready availability of inexpensive computing resources nowadays, we opt instead to develop a computer model that can accurately predict the characteristics of this N-port junction (for whatever choice of N).

Actually, the structure depicted in Fig. 1 resembles that recently analyzed by Bialkowski [1]. There are, however, two major differences. First, the structure described in [1] has only a single metallic post in the central cavity. Following the suggestion made by Montgomery *et al.* [9], we have additionally inserted a concentric dielectric sleeve, and there are thus two tuning elements at our disposal. Second, Bialkowski's analysis is based on a noneigenmode procedure, which may tend to be rather unwieldy when N becomes large. To counter this, we have elected for an eigenmode approach, which is more convenient to handle, since it permits the same software program to be used for different settings of N.

II. Development of Model

For simplicity, we shall assume, as in [1], that each of the N rectangular-waveguide arms (of dimensions a and b where $a > b$) supports only the dominant TE_{10} mode. However, unlike [1], the modes in the central cavity (of radius r_o and height a) are not of the simpler radial-TE representations employed by Bialkowski. The presence of the dielectric sleeve (of relative permittivity ε_r, outer radius r_d, inner radius r_p, and height a) entails instead the use of the more complicated hybrid-mode representations for the fields inside the junction.

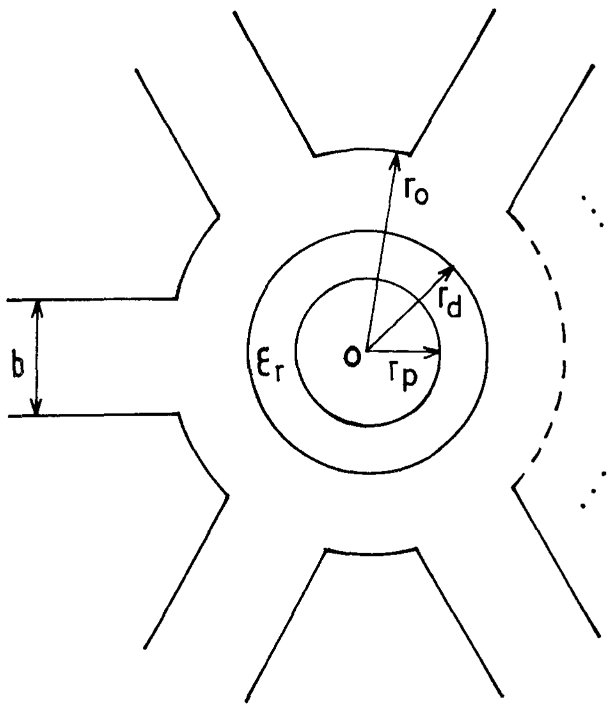

Fig. 1. Symmetrical N-port (E-plane coupled) waveguide junction with dielectric sleeve and metallic post in central cavity. Cavity center O is also origin of cylindrical (r, ϕ, z) coordinate system, and height of cavity = broad dimension of waveguide arms = a

The structure of Fig. 1 supports N eigenmodes. For the eigenmode of order k, there are N equal-magnitude inward-traveling waves within the rectangular-waveguide arms with the waves in any pair of neighboring arms having a phase difference of $\pm 2\pi k/N$. If we regard the central cavity as consisting of N adjoining sector portions (each subtending an angle of $2\pi/N$), we can then infer from the N-fold rotational symmetry of the junction that there will similarly be a phase difference of $\pm 2\pi k/N$ between the cavity fields in any pair of adjacent sectors. Hence, instead of studying the entire structure with all its N ports, we only need to analyze the fields present in any representative $2\pi/N$ portion of the junction.

It should also be noted that the various eigenmodes exist as degenerate pairs (with the exception of the $k = 0$ eigenmode

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The authors are with the Department of Electrical Engineering, National University of Singapore, Kent Ridge Campus, Singapore 0511.
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TABLE I
NUMERICAL RESULTS OBTAINED BY LSBRM MODEL FOR λ_1 OF SYMMETRICAL SIX-PORT WAVEGUIDE JUNCTION USING DIFFERENT MODE RATIOS P/Q
a = 22.9 mm, b = 10.2 mm, r_o = 13.4 mm, r_d = 5.0 mm, r_p = 1.8 mm, ε_r = 3, f = 10.0 Hz

# modes	λ_1 values obtained by using different P/Q ratios			
M	P = 2M Q = M	P = M Q = M	P = M Q = 2M + 1	P = M Q = 3M
1	0.752 ∠107.9°	0.712 ∠109.2°	0.795 ∠ 90.8°	0.795 ∠ 90.8°
3	0.910 ∠ 87.3°	0.905 ∠ 87.4°	0.953 ∠ 82.9°	0.954 ∠ 82.9°
5	0.969 ∠ 83.9°	0.967 ∠ 84.0°	0.978 ∠ 81.2°	0.981 ∠ 81.1°
7	0.978 ∠ 82.0°	0.978 ∠ 82.0°	0.988 ∠ 80.6°	0.991 ∠ 80.6°
9	0.985 ∠ 80.9°	0.985 ∠ 80.9°	0.994 ∠ 80.5°	0.995 ∠ 80.4°
11	0.990 ∠ 80.7°	0.990 ∠ 80.7°	0.995 ∠ 80.4°	0.997 ∠ 80.3°
13	0.992 ∠ 80.6°	0.992 ∠ 80.6°	0.996 ∠ 80.3°	*
15	0.994 ∠ 80.5°	0.994 ∠ 80.5°	0.997 ∠ 80.3°	*

when N is odd or the $k = 0$ and $k = \frac{1}{2}N$ eigenmodes when N is even). We are, therefore, able to economize on computing time as there is no need for us to repeat the computations for nearly half of the N eigenmodes. For each eigenmode of order k, we assign λ_k to be the input reflection coefficient looking into any of the N ports (i.e., reflected-to-incident wave ratio for the dominant mode). Once all the distinct λ_k values are known, we may then derive the scattering parameters s_{ij} of the junction via the following conversion formulas:

$$s_{ij} = \begin{cases} \dfrac{1}{N}\displaystyle\sum_{k=0}^{(N-1)/2}(2-\delta_{k,0})\lambda_k \cos\frac{2\pi k(i-j)}{N} & \text{for odd } N \\ \dfrac{1}{N}\displaystyle\sum_{k=0}^{N/2}(2-\delta_{k,0}-\delta_{k,\frac{n}{2}})\lambda_k \cos\frac{2\pi k(i-j)}{N} & \text{for even } N \end{cases} \tag{1}$$

where δ represents the Kronecker-delta function.

To obtain the value of λ_k, we need to develop an electromagnetic model of the $2\pi/N$ portion of the junction. The numerical technique we have selected for this task is the Least-Squares Boundary Residual Method (LSBRM), which is known for its mathematical rigor. Various versions currently exist [10]–[14]. The procedure we have used follows closely that outlined in [10]–[11]. The actual implementation is, unfortunately, rather involved and space restrictions do not permit us to provide a detailed description here. (*N.B.* Readers who require details of the analysis, including expressions for the modal fields, may write in to the principal author for a copy of the working paper.)

There is, however, one point that we would like to mention before proceeding to the next section. For the simpler case of no dielectric sleeve in the central cavity, Bialkowski [1] reported that he had to use certain approximations in the computation of the Bessel-function values for the TE fields in his unloaded cavity. We, in fact, have even more Bessel-function terms to compute because of the hybrid modes within our dielectric-loaded cavity. Nevertheless, there is no requirement for us to resort to the approximations employed by Bialkowski as we have found that the numerical routine recently proposed by du Toit in [15] is able to generate sufficiently accurate values for all the different Bessel-function terms we encountered in our computer program.

III. SAMPLE RESULTS

The analysis outlined in Section II has since then been implemented as a software package that is compact enough for running on the IBM 486 PC or its equivalent. Although the actual program can, in general, handle any number of ports N, the sample results we provide here are only for the $N = 6$ case (since the symmetrical six-port waveguide junction is of interest to us for another project that we are currently working on [16]).

Presented in Table I are some results obtained from our tests for "relative convergence" [17] (which is known to affect certain other numerical techniques). For Bialkowski's model [1], there is a need to ensure that the number of modes used in the cavity-field representation is $2\pi r_o/b$ times that employed for the waveguide-field representation. One of the reasons why we select the LSBRM is that Davies has already shown in [10] that this rigorously convergent numerical technique should be free from such a problem. From the tabulated LSBRM results, we observe that the converged values for both magnitude and phase of λ_1 remain the same for whatever ratio of modes P/Q (where P and Q are, respectively, the numbers of modes used in the waveguide- and cavity-field representations). Similar checks of the λ_k values obtained by using other settings of k and N have also not yielded any signs of relative convergence (but to conserve space, we chose not to include all these other results for $k \neq 1$ or $N \neq 6$ in Table I).

A careful examination of all the λ_k data we have thus far amassed has revealed that the rate of convergence for this input reflection coefficient is not necessarily the same for each of the k = 0, 1, 2, 3 eigenmodes. A single figure of merit pertaining directly to the scattering matrix **S** as a whole (instead of to λ_k of the individual eigenmodes) is therefore required. In another project [16], we have already found that the singular values σ_n of **S** for any passive N-port component must satisfy the following inequality:

$$0 < \sigma_n \leq 1 \qquad \text{for } n = 1, 2, 3, \cdots, N. \tag{2}$$

TABLE II
SINGULAR-VALUE RATIO $\sigma_{\max}/\sigma_{\min}$ VALUES OF SCATTERING MATRIX FOR SYMMETRICAL SIX-PORT WAVEGUIDE JUNCTION (USING DIFFERENT MODE RATIOS P/Q FOR LSBRM)
$a = 22.9$ mm, $b = 10.2$ mm, $r_o = 13.4$ mm, $r_d = 5.0$ mm, $r_p = 1.8$ mm, $\varepsilon_r = 3$, $f = 10.0$ GHz

# modes	$\sigma_{max}/\sigma_{min}$ of resulting S for different P/Q ratios				
M	P = M Q = M	P = 2M Q = M	P = M Q = 2M + 1	P = M Q = 3M	P = M Q = 4M - 1
1	2.187	2.313	1.265	1.264	1.264
3	1.157	1.144	1.138	1.085	1.072
5	1.145	1.117	1.050	1.045	1.039
7	1.058	1.045	1.024	1.023	1.021
9	1.040	1.037	1.018	1.013	1.009
11	1.030	1.030	1.011	1.008	*
13	1.022	1.020	1.009	*	*
15	1.015	1.017	1.007	*	*
21	1.010	*	*	*	*
25	1.007	*	*	*	*
29	1.005	*	*	*	*
33	1.004	*	*	*	*

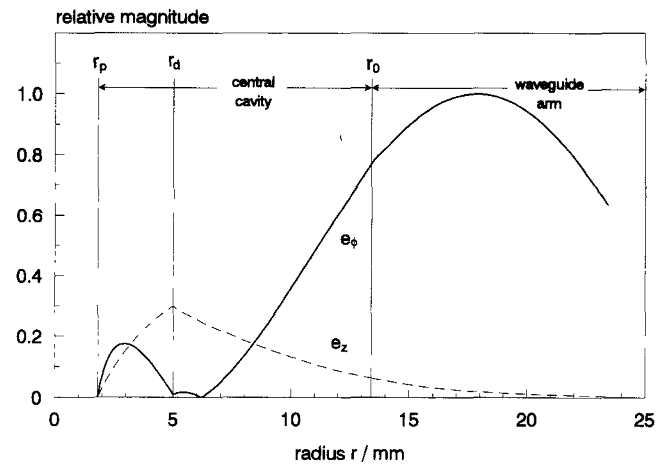


Fig. 2. Radial variations of electric field components e_ϕ and e_z predicted by LSBRM model for symmetrical six-port waveguide junction. $a = 22.9$ mm, $b = 10.2$ mm, $r_o = 13.4$ mm, $r_d = 5.0$ mm, $r_p = 1.8$ mm, $\varepsilon_r = 3$, $f = 10.0$ GHz.

For our present lossless six-port case, all six singular values ought to be equal to unity since it is obvious that the resultant S should be unitary. A single-figure parameter that we may then use to assess the overall accuracy of S is the ratio of the largest to the smallest of these six singular values, *viz* . $\sigma_{\max}/\sigma_{\min}$ (which also yields the 2-norm condition of S). The results of Table II confirm that this ratio does converge to the expected value of unity (within 0.5% at $P = Q = 33$) regardless of the ratio of modes P/Q used.

Our software package may also be used to compute the fields within the junction. The sample plot given in Fig. 2 depicts field magnitudes as a function of the radial coordinate r. It can be seen that the boundary conditions at the interfaces between metal and dielectric and between dielectric and air have been adequately met. In addition, we observe that the hybrid modes in the central cavity have an e_z component (parallel to the axis of the central cavity) which is absent in the structure analyzed by Bialkowski [1]. The magnitude of e_z decays with increasing r, and well into the rectangular-waveguide arm (i.e., for large values of r) the field pattern once again reverts to that of the nonevanescent TE_{10} mode.

Finally, we have to supplement the computational validations with experimental tests since we need to check on the validity of the assumptions that we had incorporated into the model. Plotted in Fig. 3 are the measurement results taken by the HP8510C network analyzer for the various scattering parameters of an X-band symmetrical six-port waveguide junction (with dielectric sleeve and metallic post in its central cavity). Of the 36 entries in the 6×6 scattering matrix of the junction, only 4 are distinct, and we label them as s_{11}, s_{21}, s_{31} and s_{41} in Fig. 3. Excellent agreement is achieved between the measured data points and the corresponding predicted graphs for the magnitudes [Fig. 3(a)] and phases [Fig. 3(b)] of all the scattering parameters over the entire waveguide bandwidth, thereby attesting to the suitability of the LSBRM for our present modeling task.

IV. CONCLUSION

The present paper has outlined the application of the mathematically rigorous LSBRM to the analysis of the symmetrical N-port waveguide junction. Montgomery *et al.* [9] have proposed (for their own design of the five-port star junction) the possibility of inserting into the central region two separate tuning elements that do not destroy the overall symmetry of the component. A similar strategy has been adopted here for the structure of Fig. 1, and we thus have at our disposal more independently adjustable parameters than what Bialkowski had in [1] for design optimization.

Computational and experimental tests have indicated that our resultant model is capable of yielding numerical accuracies in the region of ±0.5% for the loaded junction's scattering parameters. Plots of the field patterns within the junction can also be generated. Although the sample results we have pro-

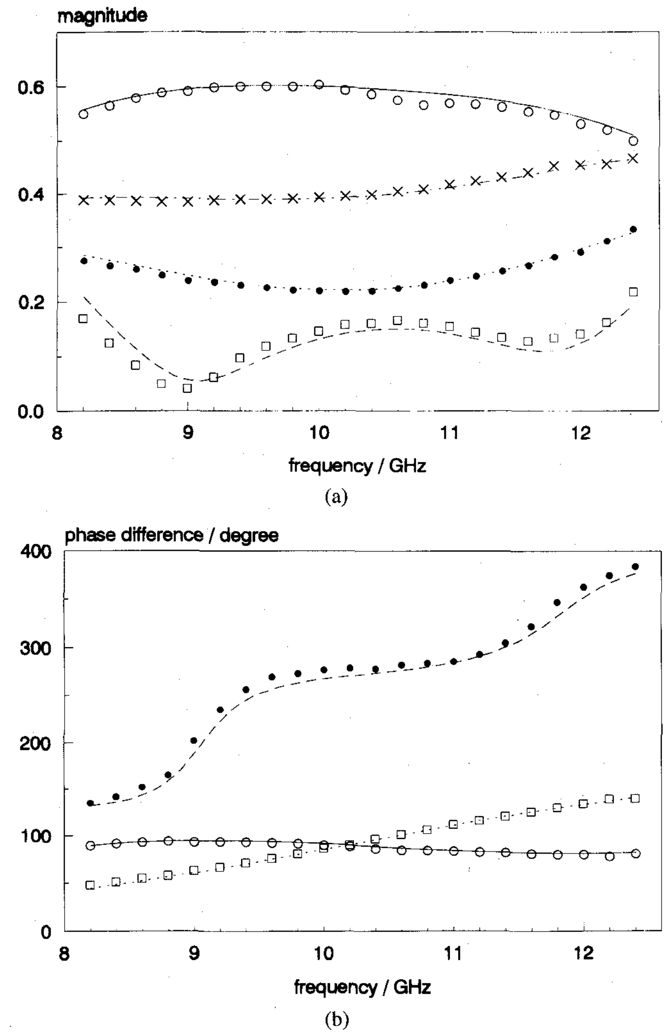


Fig. 3. (a) Variations of scattering-parameter magnitudes against frequency for symmetrical six-port waveguide junction: $|s_{11}|$: predicted ---- measured □□ □ □; $|s_{21}|$: predicted {—}{—} measured o o o o; $|s_{31}|$: predicted ------ measured • • ••; $|s_{41}|$: predicted {—} - - {—} measured × ×× ×. (b) Variations of scattering-parameter phase differences against frequency for symmetrical six-port waveguide junction: $\arg[s_{11}/s_{21}]$: predicted ---- measured • •• •; $\arg[s_{21}/s_{31}]$: predicted {—}{—} measured o o oo; $\arg[s_{31}/s_{41}]$: predicted ------ measured □ □□ □. $a = 22.9$ mm, $b = 10.2$ mm, $r_o = 14.9$ mm, $r_d = 6.5$ mm, $r_p = 5.6$ mm, $\varepsilon_r = 2$.

vided in Section III are for the symmetrical six-port waveguide junction, the model can easily handle other $N \neq 6$ settings as well. (*N.B.* Readers may write in for a complimentary copy of the software package, which runs on an IBM 486 PC or its equivalent.)

REFERENCES

[1] M. E. Bialkowski, "Analysis of an n-port consisting of a radial cavity and E-plane coupled rectangular waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 1840–1843, Sept. 1992.

[2] P. Carle, "Multi-port branch-waveguide couplers with arbitrary power splitting," in *IEEE Int. Microwave Symp. Dig.*, 1989, pp. 317–320.

[3] K. Chang, M. Li, K. A. Hummer, and R. A. Speciale, "High-power four-way power divider/combiner," in *IEEE Int. Microwave Symp. Dig.*, 1990, pp. 1329–1332.

[4] G. P. Riblet, "An E-plane coupled matched waveguide five-port for making six-port measurements over a full waveguide band," in *IEEE Int. Microwave Symp. Dig.*, 1983, pp. 372–374.

[5] M. Riaziat and G. Zdasiuk, "Waveguide star junction used in Ka-band dual six-port measurements," in *IEEE Int. Microwave Symp. Dig.*, 1985, pp. 539–594.

[6] S. P. Yeo and K. H. Lee, "Improvements in design of six-port reflectometer comprising symmetrical five-port waveguide junction and directional coupler," *IEEE Trans. Instrum. Meas.* , vol. 39, pp. 184–188, Feb. 1990.

[7] S. P. Yeo, "Analysis of symmetrical six-port junction when configured as a six-port reflectometer," *IEEE Trans. Instrum. Meas.*, vol. 41, pp. 193–197, Apr. 1992.

[8] U. Stumper, "New nondirectional waveguide multi-port coupler as part of a simple microwave six-port reflectometer," *Electron. Lett.*, vol. 18, pp. 757–758, Sept. 1982.

[9] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits.* New York: McGraw-Hill, 1948.

[10] J. B. Davies, "A least-squares boundary residual method for the numerical solution of scattering problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 99–104, Feb. 1973.

[11] A. L. Cullen and S. P. Yeo, "Using the least-squares boundary residual method to model the symmetrical five-port waveguide junction," *Inst. Elec. Eng. Proc. Microwaves Antennas Propagat.*, vol. 134-H, pp. 116–124, Apr. 1987.

[12] K. J. Bunch and R. W. Grow, "Numerical aspects of the boundary residual method," *Int. J. Numerical Modeling Electronic Networks Devices Fields*, vol. 3, pp. 57–71, Feb. 1990.

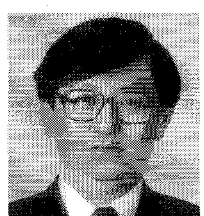
[13] M. Essaaidi and M. Boussouis, "A modified least-squares boundary residual method for the rigorous analysis of MIC's," *Microwave Opt. Technol. Lett.*, vol. 5, pp. 537–542, Sept. 1992.

[14] H. Oraizi and J. Perini, "A numerical method for the solution of the junction of cylindrical waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 640–642, Oct. 1973.

[15] C. F. du Toit, "The numerical computation of Bessel functions of the first and second kinds for integer orders and complex arguments," *IEEE Trans. Antennas Propagat.*, vol. 38, pp. 1341–1349, Sept. 1990.

[16] S. P. Yeo, M. Cheng, D. J. Bannister, and J. P. Ide, "Detection of 180° phase discrepancies in multi-port waveguide transmission measurements," in *IEEE Instrum. Meas. Technol. Conf. Dig.*, 1994, pp. 874–877.

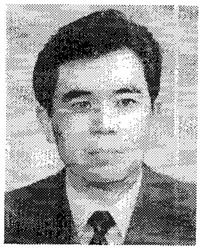
[17] R. Mittra and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves.* New York: MacMillan, 1971.



S. P. Yeo (M'80–M'87) was born in Singapore in 1956. He received the B.A. degree in electrical sciences from University of Cambridge in 1978, the M.Eng. degree in electrical engineering from National University of Singapore in 1983, and the Ph.D. degree in microwave engineering from University of London in 1985.

He was with the Ministry of Defense, Singapore, for three years (1978–1980); thereafter he joined the National University of Singapore where he is presently a senior lecturer with the Electrical Engineering Department. He was also with Hewlett Packard Company (Santa Rosa, CA, USA) during his sabbatical leave over the 1992–1993 academic year. His research interests are in microwave instrumentation and electromagnetic modeling. Three of his papers have been selected for best paper awards by the Institution of Electrical Engineers (London, UK)—two for the *Electronics Letters Premium* in 1985 and another for the *Ambrose Fleming Premium* in 1988.

Dr. Yeo has been actively serving on various IEEE committees. He is currently the Vice-Chairman of the IEEE Singapore Chapter of Microwave Theory and Techniques, Vice-Chairman of the IEEE ICCS/94 Organizing Committee, and Publicity Chairman on the IEEE GLOBECOM/95 Organizing Committee. He has also been invited to sit on the International Honorary Committee for IEEE IMTC/94.



L. Qiao was born in Beijing, R.O.C., in 1960. He received the B.S. degree in physics from Beijing Normal University, Beijing, R.O.C, in 1983 and the M.Eng. degree in electrical engineering from Northern Jiaotong University, Beijing, R.O.C, in 1986.

He remained at the Northern Jiaotong University for the next six years as a lecturer with the Department of Physics. He is currently a research scholar with the Department of Electrical Engineering, National University of Singapore, Singapore, where he is working toward his Ph.D. in electrical engineering. His research interests include measurement techniques based on six-port reflectometers as well as numerical techniques for modeling of microwave and optical structures.

M. Cheng was born in Shanghai, R.O.C., in 1963. He received both the B.S. and M.S. degrees in electrical engineering from Fudan University, Shanghai, R.O.C., in 1984 and 1987, respectively.

Upon graduation, he joined Fudan University as a staff of the Electrical Engineering Department. He is currently a research scholar with the Electrical Engineering Department, National University of Singapore, Singapore, where he is working toward his Ph.D. in electrical engineering. His research interests include microwave-measurement techniques and optical-fiber communications.